Twentyfirst Colloquiumfest (Cutkoskyfest)

In honour of the 60th birthday of
Prof. Steven Dale Cutkosky
Federal University of São Carlos - UFSCar
São Carlos–SP, Brazil

27 and 28 July, 2019

Program

	Saturday, 27th of July	Sunday, 28th of July
8:30 –8:50	Registration	
8:50 – 9:00	Opening (F.-V. Kuhlmann)	
9:00 – 9:45	D. Cutkosky	O. Kashcheyeva
9:50 – 10:35	S. Wiegand	J. Verma
10:40 – 11:10	Coffee Break	Coffee Break
11:10 – 11:55	K. Kuhlmann	H. Srinivasan
12:00 – 12:45	R. Wiegand	M. Spivakovsky
12:45 – 14:30	Lunch Break	Lunch Break
14:30 – 14:50	S. Roy	"Trip" to Parque Ecológico
14:55 – 15:15	M. Moraes	
15:20 – 15:40	D. Mondal	
15:45 – 16:15	Coffee Break	
16:15 – 17:00	A. Simis	
17:00 – 18:00	Problem session	

Contents

Program iii

Abstracts 1

Ordinary and Mixed Multiplicities of Valuative and Divisorial Filtrations (*Steven Dale Cutkosky*) 1

Key polynomials and generating sequences of valuations centered in $k[x, y, z]$ (*Olga Kashcheyeva*) 1

Real holomorphy rings in function fields and their units (*Katarzyna Kuhlmann*) 2

Finiteness of integral closures of complete local rings (*Dibyendu Mondal*) . 2

Perron Transforms and Hironaka's game (*Michael Moraes*) 2

Components of bigraded local cohomology modules (*Sudeshna Roy*) 3

On perfect ideals of codimension 2 with linear presentation (*Aron Simis*) . 3

The problems of resolution of singularities and local uniformization in arbitrary characteritic (*Mark Spivakovsky*) 4

Hilbert Coefficients of Gorenstein Algebras (*Hema Srinivasan*) 4

Product of complete ideals (*Jugal Verma*) 5

Vanishing of Tor over fiber products (*Sylvia Wiegand*) 5

Torsion in the tensor product of a module and its dual (*Roger Wiegand*) . 6

Speaker Index 7

Abstracts

Ordinary and Mixed Multiplicities of Valuative and Divisorial Filtrations

Steven Dale Cutkosky

University of Missouri, USA

We define mutiplicities and mixed multiplicities of the (generally non-Noetherian) filtrations associated to valuations and divisors on an excellent local domain. We show that they enjoy many of the good properties of multiplicities and mixed multiplicities of m-primary ideals. In particular, we show that Rees's theorem showing that m-primary ideals I contained in J have the same multiplicity if and only if they have the same integral closure extends to the case of inclusions of filtrations of valuations and divisors.

Key polynomials and generating sequences of valuations centered in $k[x, y, z]$

Olga Kashcheyeva

University of Illinois at Chicago, USA

For a rational rank 1 valuation ν centered in $k[x, y, z]$ we will discuss two separate algorithms of constructing sequences of polynomials that carry information about ν. One algorithm provides an explicit description of the first part of the sequence of key polynomials for the valued field extension $k(x, y) \hookrightarrow k(x, y, z)$. In the situation when ν does not posses a limit key polynomial it provides a complete set of key polynomials. The other algorithm produces a much bigger set of polynomials that forms a generating sequence for ν and therefore contains full information about ν.

Real holomorphy rings in function fields and their units

Katarzyna Kuhlmann
University of Szczecin, Poland

For a given formally real function field $F|K$, where K is real closed, we give a new description of the real holomorphy ring $H(F)$ of F and the relative holomorphy ring $H(F|K)$. This is based on the observation that the real places of F that are composite with the natural place of K lie dense in the space of all $\mathbb{R}$-places of F. The totally positive units of $H(F)$ are sums of squares of totally positive units of $H(F)$ as was shown by J. Schmid, and the length of such presentations is bounded by $P(F)+1$, where $P(F)$ is the Pythagorean number of F. It is not known wheather it can be reduced to $P(F)$. We will present some partial results and questions connected with this open problem.

The talk is based on joint work with F.-V. Kuhlmann and E. Becker.

Finiteness of integral closures of complete local rings

Dibyendu Mondal
IIT Bombay, India

If $A \subseteq B$ are affine domains over a field k, it is well-known that the integral closure of A in B is a finite A-module. However, no analogous result is known in the set-up of complete local rings. In this talk, we will show that if $A \subseteq B$ is an inclusion of complete local domains over C, the field of complex numbers, then the integral closure of A in B is a finite A-module, provided the induced map from Spec(B) to Spec(A) is surjective.

Perron Transforms and Hironaka's game

Michael Moraes
ICMC-USP, Brazil

In this talk we present a matricial result that generalizes Hiron- akas game and Perron transforms simultaneously. We also show how one can deduce the various forms in which the algorithm of Perron appears in proofs of local uniformization from our main result.

This is a joint work with J. Novacoski.

Components of bigraded local cohomology modules

Sudeshna Roy

IIT Bombay, India

Let C be a regular ring containing a field K of characteristic zero. Using the theory of $\mathcal{D}$-modules, in a recent work T. J. Puthenpurakal did a comprehensive study of components of graded local cohomology modules $H^i_J(S)$ where J is an arbitrary homogeneous ideal in the standard graded ring $S = C[Z_1, \ldots, Z_l]$ with $\deg c = 0$ for all $c \in C$ and $\deg Z_t = 1$ for all t.

Let $R = C[X_1, \ldots, X_n, Y_1, \ldots, Y_m]$ with $n, m \geq 1$, and let

$$A_{n+m}(C) = R\langle \partial_1, \ldots, \partial_n, \delta_1, ..., \delta_m \rangle$$

be the $(n+m)^{th}$ Weyl Algebra over C, where $\partial_i = \partial/\partial X_i$ and $\delta_j = \partial/\partial Y_j$ for all i, j. Consider R and $A_{n+m}(C)$ as bi-graded with $\text{bideg} c = (0,0)$ for all $c \in C$, $\text{bideg} X_i = (1,0)$, $\text{bideg} Y_j = (0,1)$, $\text{bideg} \partial_i = -(1,0)$ and $\text{bideg} \delta_j = -(0,1)$ for all i, j. Let I be a bihomogeneous ideal in R. The aim of this talk is to present some results of bigraded local cohomology modules $H^i_I(R)$ which are analogues of the above-mentioned work. For this purpose we introduce a class of bigraded $A_{n+m}(C)$-modules, called bigraded generalized Eulerian $A_{n+m}(C)$-modules.

This is a joint work with R. Bhattacharyya, T. J. Puthenpurakal and J. Singh.

On perfect ideals of codimension 2 with linear presentation

Aron Simis

Federal University of Pernambuco - UFPE, Brazil

The goal is the fine structure of the ideals in the title, with emphasis on the properties of the associated Rees algebra and the special fiber. The watershed between the present approach and some of the previous work in the literature is that here one does not assume that the ideals in question satisfy the usual generic properties. . One applies to three important models: linearly presented ideals of plane fat points, reciprocal ideals of hyperplane arrangements and linearly presented monomial ideals.

This is joint work with A. Doria and Z. Ramos.

The problems of resolution of singularities and local uniformization in arbitrary characteritic

Mark Spivakovsky

University of Toulouse and CNRS, France

The problem of resolution of singularities asks whether, given an algebraic variety X over a field, there exists a non-singular algebraic variety X' and a proper map $X' \to X$ which is one-to-one over the non-singular locus of X. If we cover X' by affine charts, the problem becomes one of *parametrizing* pieces of X by small pieces of the Euclidean space k^n. This local version of the problem, called Local Uniformization, is usually stated in terms of valuations and can be interpreted as follows. Let (R, M, k) be a local quasi-excellent noetherian domain (resp. a local k-algebra essentially of finite type without zero divisors) and let R_ν be a valuation ring containing R and having the same field of fractions as R. Find a *smooth* finite type R-algebra R' such that $R' \subset R_\nu$. The **Local Uniformization Theorem** asserts the existence of such an R'; it waas proved by O. Zariski in 1940 in the case when *char* $k = 0$ and is one of the central open problems in the field when *char* $k = p > 0$.

To study local uniformization we will introduce the notion of key polynomials associated to a simple extension $\iota : K \to K(x)$ of valued fiels, defined by Saunders Mac Lane in the 1930-ies in the case of discrete valuations and generalized by M. Vaquié, F.J. Herrera Govantes, J. Decaups, W. Mahboub, M. A. Olalla Acosta, J. Novacoski and M. Spivakovsky.

At the end of the talk we will discuss the applications of key polynomials to the problem of Local uniformization in arbitrary characteristic and, in particular our algorithm for monomializing the first limit key polynomial by blowings up.

Hilbert Coefficients of Gorenstein Algebras

Hema Srinivasan

University of Missouri, USA

Given a standard graded algebra S and $R = S/I$, a Cohen-Macaualay graded algebra, the Hilbert polynomial $P_R(x)$ of R can be written as $P_R(x) = \sum_{i=0}^{d-1} (-1)^i e_i \binom{x+d-i-1}{x}$. We discuss some bounds for the coefficients e_i.

For all Gorenstein algebras, e_0, also called the multiplicity, is bounded above by $e(R) \leq \frac{1}{s!} \prod_{i=1}^{k} \min\{M_i, \lfloor \frac{m_s}{2} \rfloor\} \prod_{i=k+1}^{s} \max\{m_i, \lceil \frac{m_s}{2} \rceil\}$. Here, m_i and M_i are the minimal and maximal shifts in a graded S resolution of R.

When R is Gorenstein and the resolution is quasi pure, all the e_i satisfy

$$\begin{aligned} f_l(m_1 \dots m_k M_{k+1} \dots M_s) \tfrac{m_1 \dots m_k M_{k+1} \dots M_s}{(s+l)!} &\leq e_l(S) \\ &\leq f_l(M_1 \dots M_k m_{k+1} \dots m_s) \tfrac{M_1 \dots M_k m_{k+1} \dots m_s}{(s+l)!} \end{aligned}$$

where $f_l(y_1, \dots, y_s) = \sum_{1 \leq i_1 \leq \dots i_l \leq s_l} \prod_{t=1}^{l} (y_{i_t} - (i_t + t - 1))$, $1 \leq l \leq s$ and $f_0 = 1$.

In the non quasi pure case, first coefficient, e_1 also satisfies a similar upper bound. The analogous lower bound is not satisfied in general. However, we will discuss what may be a possible lower bound and the obstacles to extending these bounds to higher coefficients.

Product of complete ideals

Jugal Verma

IIT Bombay, India

We shall survey the results of Zariski, Rees, Lipman, Reid-Roberts-Vituli, Cutcosky and Sarkar-Verma about the product of complete ideals in regular local rings, pseudo-rational local rings and polynomial rings. We will show how joint reductions and local cohomology of Rees algebras can be used to unify these results about the product of complete ideals.

Vanishing of Tor over fiber products

Sylvia Wiegand

University of Nebraska-Lincoln, USA

Let $(S, \mathfrak{m}, k)$ and $(T, \mathfrak{n}, k)$ be local rings with $S \neq k$ and $T \neq k$, and let R denote their fiber product over their common residue field k. Inspired by the work of Saeed Nasseh and Sean Sather-Wagstaff, we explore consequences of vanishing of $\mathrm{Tor}_m^R(M, N)$ for small values of m, where M and N are finitely generated R-modules. For example, we show: If $\mathrm{Tor}_i^R(M, N) = 0 = \mathrm{Tor}_j^R(M, N)$, for some odd integer $i \geq 5$ and some even integer $j \geq 6$, then $\mathrm{pd}_R M \leq 1$ or $\mathrm{pd}_R N \leq 1$.

This is a joint work with T.H. Freitas, V. H. Jorge Pérez and R. Wiegand.

Torsion in the tensor product of a module and its dual

Roger Wiegand
University of Nebraska-Lincoln, USA

More than 25 years ago, Craig Huneke and I made various conjectures on the existence of torsion in tensor products. Here is one such conjecture:

Conjecture. *Let* $(R, \mathfrak{m})$ *be a Noetherian local ring and* M *a finitely generated* R*-module with positive rank. If* $M \otimes_R M^*$ *is a maximal Cohen-Macaulay (MCM)* R*-module, then* M *is free.*

Here M^* is the algebraic dual $\mathrm{Hom}_R(M, R)$. Saying that M has positive rank just means that $Q \otimes_R M$ is a non-zero free Q-module, where Q is the total quotient ring (obtained by inverting the non-zerodivisors of R). The assumption that M has rank rules out trivial counterexamples such as $R = \mathbb{Q}[[x, y]]/(xy)$ with $M = R/(x)$. We shall concentrate on the case (still wide open) where R is one dimensional and Gorenstein and I is an $\mathfrak{m}$-primary ideal. In this context, one has $I \otimes_R I^*$ MCM (equivalently, torsion-free) if and only if I is *rigid*, that is $\mathrm{Ext}^1_R(I, I) = 0$.

We show, assuming I is rigid, that I must be principal if R has multiplicitiy at most 8 (at most 10 if R is a complete intersection). Also, if I is rigid and the singularity (R, I) is smoothable, then I must be principal.

This is joint work with C. Huneke and S. Iyengar.

Speaker Index

Cutkosky
Steven Dale, 1

Kashcheyeva
Olga, 1
Kuhlmann
Katarzyna, 2

Mondal
Dibyendu, 2
Moraes
Michael, 2

Roy
Sudeshna, 3

Simis
Aron, 3
Spivakovsky
Mark, 4
Srinivasan
Hema, 5

Verma
Jugal, 5

Wiegand
Roger, 6
Sylvia, 5